

# Average value of available measurements of the absolute air-fluorescence yield

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## Abstract

The air-fluorescence yield is a key parameter for determining the energy scale of ultra-high-energy cosmic rays detected by fluorescence telescopes. A compilation of the available measurements of the absolute air-fluorescence yield normalized to its value in photons per MeV for the 337 nm band at given pressure and temperature has been recently presented in [1]. Also, in that paper, some corrections in the evaluation of the energy deposited in the corresponding experimental collision chambers have been proposed. In this note this comparison is updated. In addition, a simple statistical analysis is carried out showing that our corrections favor the compatibility among the various experiments. As a result, an average value of 5.45 ph/MeV for the fluorescence yield of the 337 nm band (20.1 ph/MeV for the spectral interval 300 – 420 nm) at 1013 hPa and 293 K with an uncertainty of 5% is found. This result is fully compatible with that recently presented by the AIRFLY collaboration (still preliminary) in such a way that including this latest result could even lowered the final uncertainty below the 5% level with high reliability.

The fluorescence technique has been proved to be very fruitful for the study of ultra-high-energy cosmic rays. The air-fluorescence yield, defined as the number of fluorescence photons per unit deposited energy, is a key calibration parameter of the fluorescence telescopes. In [1], we presented a comparison between the laboratory measurements of the absolute air-fluorescence yield available in the literature normalized to common units (photons/MeV), air conditions (dry air, 800 hPa and 293 K) and wavelength interval (337 nm band). From the comparison of this normalized  $Y_{337}$  parameter, discrepancies larger than the quoted uncertainties of the measurements were found. According to the Monte Carlo analysis carried out in [1], non-negligible corrections in the evaluation of the energy deposition should be applied to these results, in particular in experiments where authors neglected the energy deposited outside the field of view of the light detector. In fact, the proposed corrections are even larger than the quoted uncertainties in some cases (i.e., measurements of Kakimoto *et al.* [2] and Lefeuve *et al.* [3]). As a result of these corrections, the  $Y_{337}$  values get closer to each other in such a way that they are contained within the 6 – 7 ph/MeV interval at the above reference pressure and temperature [1].

In this work, the calculations presented in [1] have been updated and the compatibility of the fluorescence yield results with and without our corrections has been evaluated quantitatively. This has allowed to obtain a reliable  $Y_{337}$  value resulting from an average of the available absolute measurements. With respect to the MC analysis performed in [1], some improvements in the algorithm for the evaluation of the energy deposited in the collision chamber of the experiments have been made. In particular, the density correction on the cross section for the ionization processes leading to ejection of K-shell electrons, which was neglected in [1], have been included in this work. These highly excited ions generate X rays which deposit all their energy within a very short distance. As a consequence, the density correction for the K shell lowers the energy deposited by high-energy electrons (GeV range). GEANT4 simulations of energy deposition in air carried out by MACFLY [4] and AIRFLY [5] are in agreement with our calculations at the level of 2% (1%) for electron energies in the GeV (MeV) range.

In table 1, we present the compilation of fluorescence yield values [1] with these updated corrections<sup>1</sup>. In this case the fluorescence yield have

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<sup>1</sup>The updated corrections are very similar to those proposed in [1] except for experiments working with very-high-energy electrons; however, the average fluorescence yields

Table 1: Comparison of available measurements of the absolute air-fluorescence yield normalized to the 337 nm band at 1013 hPa and 293 K (dry air). Experiments are listed in column 1, the electron energies are shown in column 2 and the corresponding results, both uncorrected and corrected (in bold), are listed in column 3. The experimental uncertainties quoted by the authors are shown in column 4 and the size of the proposed correction is displayed in the last column.

Experiment	$E$ (MeV)	$Y_{337}$ (ph/MeV)	Quoted error	Correction
Kakimoto [2]	1.4	4.5 / <b>4.8</b>	10%	+6%
	300	4.4 / <b>5.5</b>		+25%
	650	3.8 / <b>4.8</b>		+27%
	1000	4.3 / <b>5.5</b>		+29%
Nagano [7]	0.85	5.0 / <b>5.4</b>	13%	+6%
Lefeuve [3]	1.1	5.1 / <b>5.5</b>	5%	+7%
	1.5	5.6 / <b>6.1</b>		+8%
MACFLY [4]	1.5	4.3 / <b>4.4</b>	13%	+1%
	$20 \cdot 10^3$	4.4 / <b>4.3</b>		-2%
	$50 \cdot 10^3$	4.6 / <b>4.5</b>		-2%
FLASH [8]	$28.5 \cdot 10^3$	5.5 / <b>5.6</b>	7.5%	+2%
AirLight [9]	0.2 – 2	5.8 / <b>5.4</b>	16%	-7%
AIRFLY [6]	$120 \cdot 10^3$	5.6 / -	$\lesssim 5\%$	-

been normalized to its value at 1013 hPa (instead of 800 hPa as presented in [1]) and 293 K. The latest result of AIRFLY [6] carried out using 120 GeV protons have been included in this compilation, but the impact on the average has been studied separately, as this value is still preliminary.

The fluorescence yield results have been displayed as a function of energy in figures 1 (uncorrected values) and 2 (corrected ones). As can be appreciated, they are in a better agreement when including our corrections. In addition, the corrected results give more support to the expected energy independence of the fluorescence yield.

In order to know quantitatively to what extent our corrections favor the agreement between the absolute results included in this comparison, a statistical analysis presented below are basically not affected by this update.

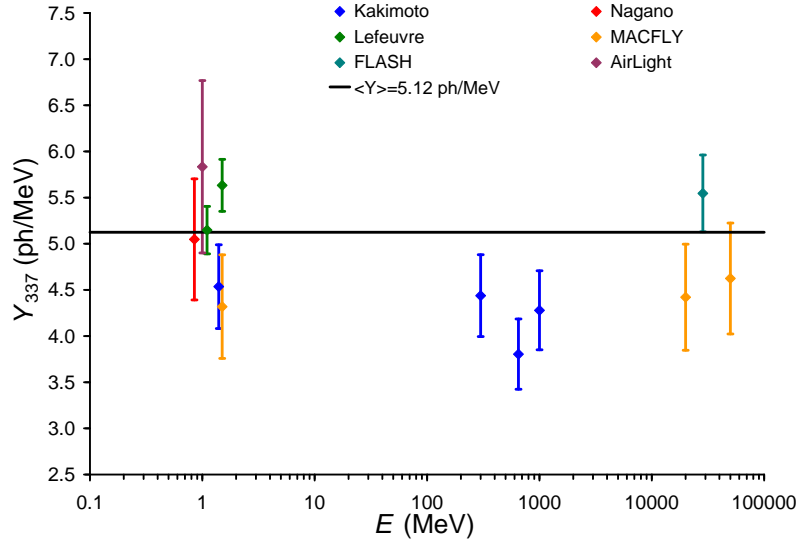


Figure 1: Comparison of uncorrected normalized  $Y_{337}$  values as a function of energy. The horizontal line represents the corresponding weighted average value (see text for details).

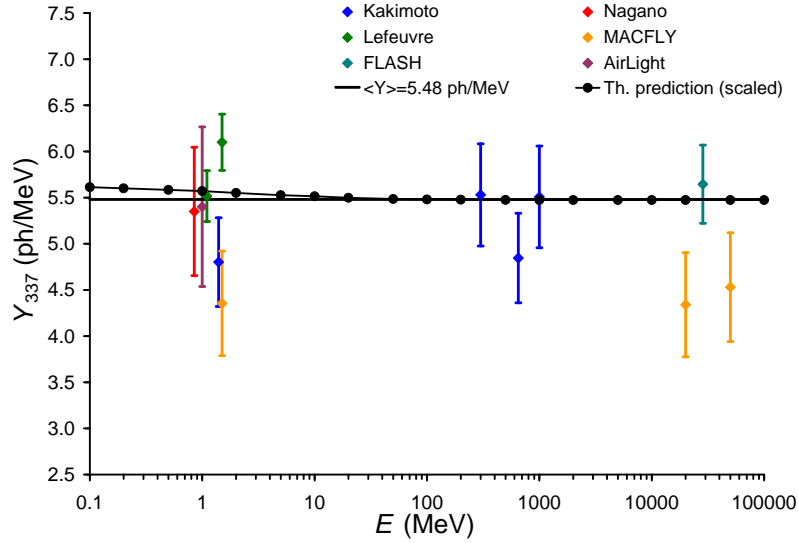


Figure 2: Same as figure 1 for the corrected  $Y_{337}$  values. The black circles connected by lines represent the weak energy dependence predicted by our simulation [10].

Table 2: Comparison of absolute  $Y_{337}$  values of table 1, where measurements reported by a given experiment at different electron energies have been averaged.

Experiment	$Y_{337}^{\text{uncorr}}$ (ph/MeV)	$Y_{337}^{\text{corr}}$ (ph/MeV)	Quoted error
Kakimoto [2]	4.3	5.2	10%
Nagano [7]	5.0	5.4	13%
Lefevre [3]	5.4	5.8	5%
MACFLY [4]	4.5	4.4	13%
FLASH [8]	5.5	5.6	7.5%
AirLight [9]	5.8	5.4	16%
AIRFLY [6]	5.6	-	$\lesssim 5\%$

tical analysis has been performed. In the first place, for a given experiment, results obtained at different energies have been averaged assuming a common systematic error<sup>2</sup>. The results are shown in table 2 for both the uncorrected (column 2) and corrected (column 3)  $Y_{337}$  values together with the percentage uncertainty associated to each experiment (column 4).

In the second place, the weighted average of the data sample shown in table 2 has been computed from

$$\langle Y \rangle = \frac{\sum_i w_i Y_i}{\sum_i w_i}, \quad (1)$$

using as weights the reciprocal of the quoted square uncertainties (i.e.,  $w_i = 1/\sigma_i^2$ ). If these  $\sigma_i^2$  are assumed to actually represent the variances of the corresponding (normal) probability distributions, our weighted mean would be the best estimator of the  $Y_{337}$  value with a variance given by  $(\sum_i 1/\sigma_i^2)^{-1}$ . However, note that authors usually do not include any error contribution from the evaluation of the deposited energy, and thus, uncertainties are very likely underestimated, at least in some experiments. In fact, the  $\chi^2$  statistic divided by the number of degrees of freedom ( $\chi^2/\text{ndf}$ ) is found to be somewhat larger than 1 for the uncorrected sample. Therefore, following the usual

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<sup>2</sup>This way, no higher significance is given to experiments measuring at several energies than those measuring at a single energy.

procedure, the variance of the weighted mean was corrected multiplying by  $\chi^2/\text{ndf}$ , that is,

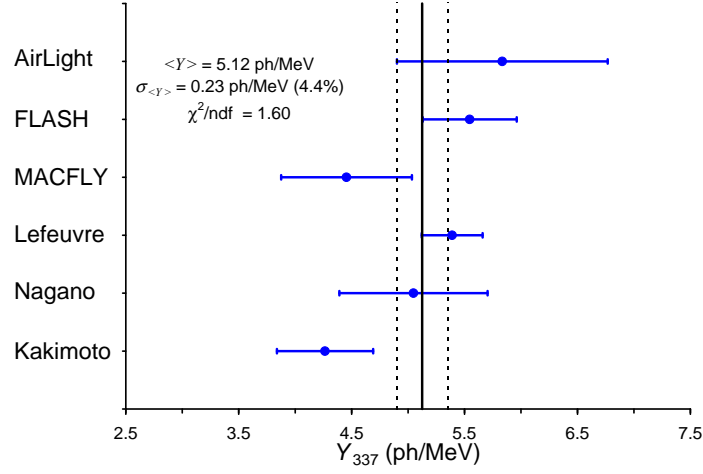
$$\sigma_{\langle Y \rangle}^2 = \frac{\chi^2/\text{ndf}}{\sum_i 1/\sigma_i^2} = \frac{\sum_i w_i (Y_i - \langle Y \rangle)^2}{(n-1) \sum_i w_i}. \quad (2)$$

Data listed in table 2 for both the uncorrected and the corrected sample are plotted in figure 3. The average value for the uncorrected  $Y_{337}$  sample (figure 3a) is found to be  $\langle Y \rangle = 5.12$  ph/MeV with  $\sigma_{\langle Y \rangle} = 0.23$  ph/MeV (4.4%) and a  $\chi^2/\text{ndf}$  value of 1.60, whereas the corresponding results for the corrected sample (figure 3b) are  $\langle Y \rangle = 5.48$  ph/MeV,  $\sigma_{\langle Y \rangle} = 0.20$  ph/MeV (3.6%) and  $\chi^2/\text{ndf} = 1.07$ . Therefore, our corrections lead to a more consistent data sample suggesting that they do improve the determination of the deposited energy for the different experiments.

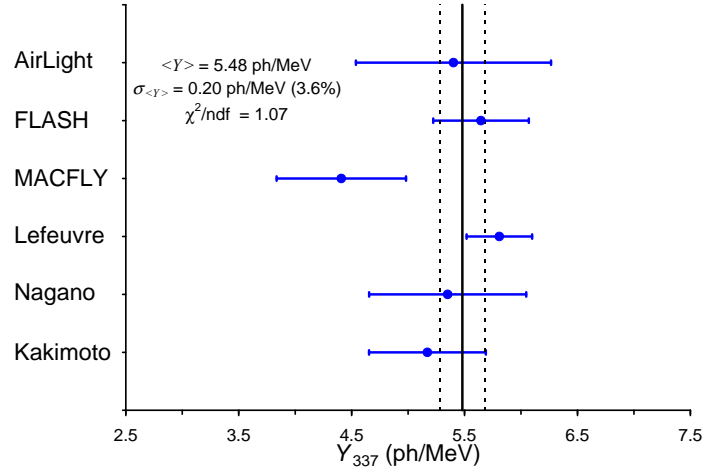
In the absence of other objective criteria, data have been weighted using the quoted experimental uncertainties, as mentioned above. Nevertheless, this weighting procedure has no significant effect on the final result. For instance, assuming the same weight (i.e., the same variance) for all the experiments the results are  $\langle Y \rangle = 5.09$  ph/MeV and  $\sigma_{\langle Y \rangle} = 0.25$  ph/MeV (5.0% error) for the uncorrected data sample and  $\langle Y \rangle = 5.30$  ph/MeV and  $\sigma_{\langle Y \rangle} = 0.20$  ph/MeV (3.8% error) for the corrected one. The weighted sample mean turns out to be somewhat larger than the non-weighted mean mainly due to the high statistical significance of the results of Lefeuvre *et al.* and FLASH, with the smallest associated uncertainties. In any case, a similar decrease in the standard deviation is found when corrections are applied whichever weights are chosen.

As a further test, it has been checked that the weighted mean of the corrected data sample does not change significantly if some measurement is excluded. For instance, removing the MACFLY result, which is the one showing the largest departure from the average, would lead to 5.61 ph/MeV. Instead, excluding the result of Lefeuvre *et al.*, with a claimed uncertainty of 5% that is incompatible with our correction of 7%, an average fluorescence yield of 5.23 ph/MeV is obtained.

We have also studied the energy (in)dependence of the fluorescence yield in this data sample. Several experiments have supported the assumed independence of the fluorescence yield at the level of about 10% in several energy intervals. On the other hand, the theoretical analysis described in [10] predicts a slight increase ( $\sim 2\%$ ) with decreasing energies in the 0.1 – 10 MeV



(a) Uncorrected  $Y_{337}$  values.



(b) Corrected  $Y_{337}$  values.

Figure 3: Graphical representation of the  $Y_{337}$  values at 1013 hPa and 293 K (see table 2). The weighted mean value  $\langle Y \rangle$  (vertical continuous line), the standard deviation of the mean  $\sigma_{\langle Y \rangle}$  (half of interval between dashed lines) and the  $\chi^2$  statistic normalized by the number of degrees of freedom are shown in the legends. (a) Discrepancies between the uncorrected values are sometimes larger than quoted errors, resulting in a relatively large  $\chi^2/\text{ndf}$  value. (b) Better agreement and consistency of the data sample are found when applying the proposed corrections.

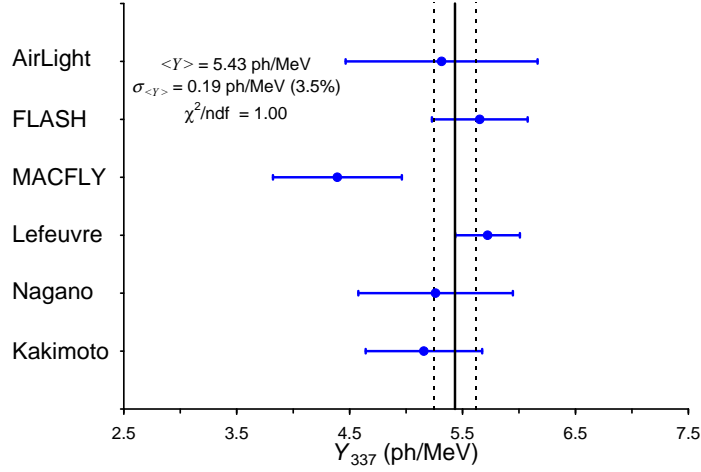


Figure 4: The same as figure 3b but previously scaling to 100 MeV using the weak energy dependence predicted by our simulation (see figure 2). The consistency of the data sample is strengthened slightly further.

range, which is also compatible with experimental data (see figure 2). In principle, this effect should be included in order to quantify the consistency of the available results. Average values listed in table 2 have been recalculated by previously scaling all the measurements to a common electron energy of 100 MeV according to the energy dependence predicted by our simulation. The results are shown in figure 4. This energy scaling slightly lowers the  $\chi^2/\text{ndf}$  value (from 1.07 down to 1.00) while both the average value and its uncertainty remain nearly unchanged (from  $5.48 \pm 0.20$  to  $5.43 \pm 0.19$  ph/MeV). Although this result would support the weak energy dependence predicted by our simulation, we note that the evidence is still very weak and this small reduction of  $\chi^2/\text{ndf}$  after the energy correction might have happened by chance. Therefore, even if confirmed experimentally<sup>3</sup>, this weak energy dependence would not have any relevant impact on the energy reconstruction of cosmic rays using the fluorescence technique.

Taking into account all the above considerations, including the possible effects of the weak energy dependence of the fluorescence yield and the weighting procedure, a conservative result from our analysis is  $Y_{337} = 5.45$  ph/MeV

<sup>3</sup>Currently not possible due to the accuracy limitations in the determination of the energy dependence of the fluorescence yield in this large range.



with an estimated error of 5%. According to the comparison of our simulation of the energy deposition with GEANT4, a small systematic uncertainty of below 2% should be added, although it does not affect the found improved compatibility of results (i.e., the  $\chi^2$  values). The recent absolute measurement of the AIRFLY collaboration [6] yields  $Y_{337} = 5.6$  ph/MeV with an uncertainty of  $\lesssim 5\%$  (still preliminary), which is fully compatible with the above value. If this new result is included in the average, then a weighted mean value of 5.52 ph/MeV is obtained from (1) with an uncertainty of  $\lesssim 5\%$ .

As already mentioned, for the comparison presented here we have normalized the air-fluorescence yield measurements to its value for the 337 nm band at 1013 hPa and 293 K. However, in some occasions it might be more convenient to use the integral of the fluorescence yield in a wider spectral range and/or other pressure and temperature conditions. The conversion can be easily done following the procedure described in detail in [1, 10]. For instance, the above average value would be of 20.1 ph/MeV ( $\pm 5\%$ ) for the 300 – 420 nm spectral range at the same reference pressure and temperature, which would become 20.3 ph/MeV if the measurement of AIRFLY is included.

Our simulation can also provide a theoretical value of the air-fluorescence yield. Unfortunately, the evaluation of the fluorescence emission cannot be very precise due to the large uncertainties in the relevant molecular parameters. Therefore, we expect a large uncertainty in this calculation of the fluorescence yield, which we estimated to be about 25% [10]. Nevertheless, a result<sup>4</sup> for  $Y_{337}$  of 6.3 ph/MeV has been found, which is consistent with the experimental ones, providing a valuable theoretical support to these measurements.

In summary, the corrections to the measurements of the absolute air-fluorescence yield discussed in [1] increase significantly the compatibility of the various experimental results. An average value of  $Y_{337} = 5.45$  ph/MeV with a 5% uncertainty has been obtained. If the absolute fluorescence yield and error of AIRFLY are confirmed, a consensus on this important parameter with an uncertainty below the 5% level could be reached with high reliability.

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<sup>4</sup>Using the quenching parameter of [11].

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